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The Predicament of the Tunisian Economy, the Slow-Down in Factors' Productivity

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Abstract. Several studies and the mere examination of data provided by National Accounts point to a severe downward change in the economic growth of the Tunisian economy, mainly in the manufacturing sector. The aim of the paper is to perform econometric estimations of the production function, at the aggregate level and for the main sectors and sub-sectors to exhibit a possible change in the global productivity of factors of production, labor and capital. Near-evenly, i.e. with the exception of few sectors or sub-sectors, the breakdown starts from 2013 and various factors, economic and political, may explain it. The topic of the paper is to measure it, not to give explanations. What is at stake is the economic interpretation, either a downturn in the productivity of factors, or a decrease in their utilization. Answering to this question is not easy because there does not exist any survey on the rate of utilization of factors, though indirect methods can be contemplated. This issue, and the related one of the marginal productivity of capital, remain to be assessed.

Keywords. Aggregate production function, Total factors productivity, Marginal productivity, and Tunisian economy

Introduction: a brief overview of the performances of the Tunisian economy

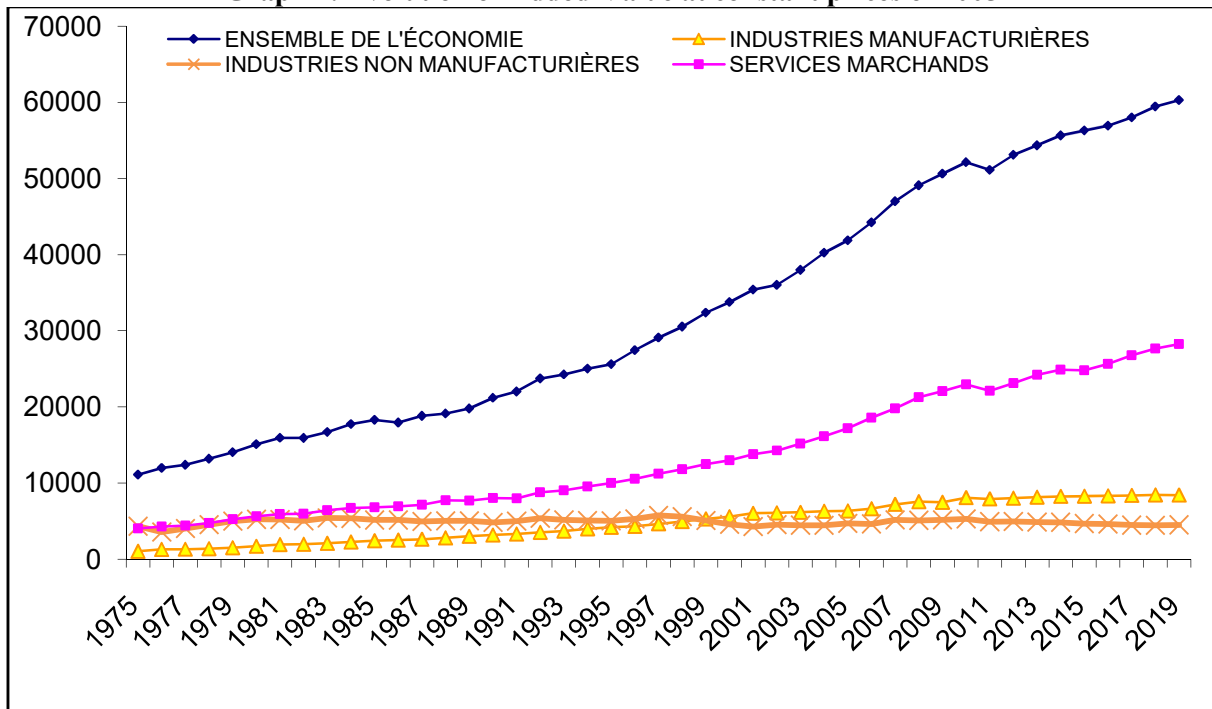
Tunisia has enjoyed during the years 1995 to 2010 a strong growth of its economy coupled with a strong increase in the factors' productivity, capital and labor. Since the Jasmine Revolution of 2011 and a tragic succession of political and economic crises the performances of the country have declined and hamper for the present time the ambitions and the social sustainability of the first and up to now only model of democracy in the Arab world.

The foundations of the economy, a well-educated labor-force and the existence of modern public infrastructures and private productive capital, and an overture to foreign trade are still in place and deserve to be mobilized for the development of the country and the well-being of the population.

Starting from 1975¹, the overview of the Tunisian economy can clearly distinguish three periods: from 1975 to 1995, 1995 to 2010 and from then up to 2019. The Table in the following page reports for the whole economy and the main sectors and sub-sectors.

The three graphs below show the corresponding yearly evolutions of added-value and production factors that materialize the change in the dynamics after 2010.

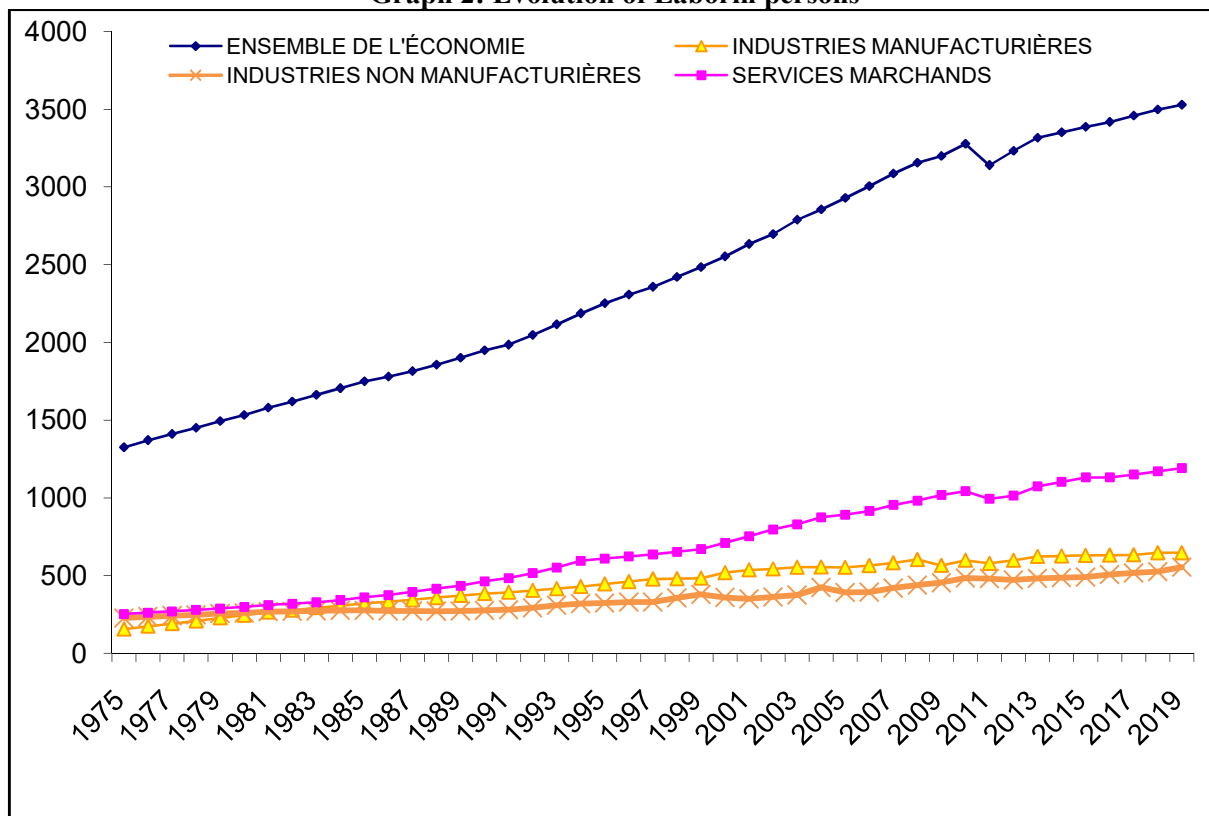
Graph 1: Evolution of Added-Value at constant prices of 2005



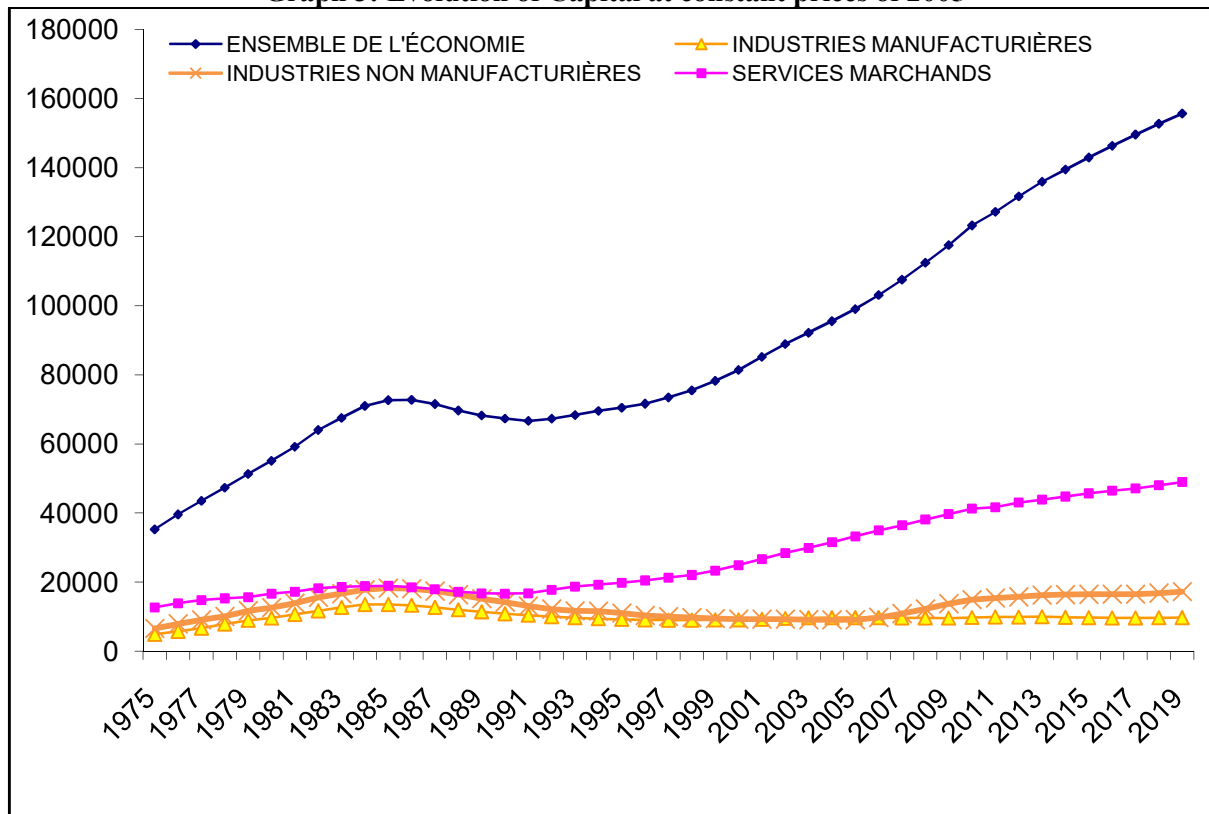
¹ Overall statistical information is only available from this year

	Period 1975 to 1995			Period 1995 to 2010			Period 2010 to 2019		
	Average rate of growth of			Average rate of growth of			Average rate of growth of		
	Added-value	Labor	Capital	Added-value	Labor	Capital	Added-value	Labor	Capital
AGRICULTURE & PÊCHE	1,5%	0,0%	3,9%	5,7%	2,0%	4,9%	3,6%	-2,0%	1,3%
INDUSTRIES MANUFACTURIÈRES	7,2%	5,4%	3,2%	6,8%	3,0%	0,6%	0,5%	0,8%	-0,1%
Industries Agricoles & Alimentaires	2,3%	7,7%	3,0%	7,4%	3,6%	2,2%	1,8%	3,4%	1,3%
Industries Mécaniques & Électriques	9,1%	9,5%	3,6%	13,7%	8,5%	2,1%	1,6%	3,0%	0,6%
INDUSTRIES NON MANUFACTURIÈRES	0,8%	1,8%	2,6%	0,5%	4,1%	3,1%	-1,8%	1,5%	1,6%
SERVICES MARCHANDS	4,7%	4,5%	2,3%	8,6%	5,5%	7,6%	2,3%	1,5%	1,9%
ADMINISTRATION	5,2%	3,1%	3,8%	8,2%	3,7%	8,9%	3,2%	1,6%	5,7%
ENSEMBLE DE L'ÉCONOMIE	4,3%	2,7%	3,5%	7,4%	3,8%	5,7%	1,6%	0,8%	2,6%
ACTIVITÉS MARCHANDES	4,5%	2,6%	2,8%	7,9%	3,8%	5,1%	1,9%	0,7%	1,5%
ACTIVITÉS MARCHANDES NON AGRICOLES	5,2%	3,9%	2,6%	8,3%	4,4%	5,1%	1,6%	1,3%	1,6%

Graph 2: Evolution of Laborin persons



Graph 3: Evolution of Capital at constant prices of 2005



A first approach to the assessment in the change in factors' productivity is to compare the change in growth in value-added to the corresponding changes in labor and capital. From the

two 1995-2010 and 2010-2019 periods the average growth in value-added of the total economy lost nearly 5 points while production factors lost approximately 3 points, exhibiting a gap of around 2 points. The same observation can be made for total market sectors and non-agricultural market sectors.

A similar scheme appears for manufacturing industries and commercial services, while the evolution of agriculture and fishing and of non-manufacturing industries are somehow different. Concerning the former the decrease in the rate of growth of production is around 2% while rates of growth of factors is substantial (4% for labor and 3.6% for capital).

As for non-manufacturing industries it is a slow-growth and now declining sector, including heterogeneous activities which deserve a detailed analytical analysis.

Agriculture and fishing appears as an exception: the period 2010-2019 exhibits a decrease in the average rate of growth of production of 2.1% compared to the period 1995-2010 but which is smaller than the decrease in the growth of factors, 4% for labor and 3.6% for capital.

An econometric approach through estimation of production functions: methodology

A detailed and precisely-dated measurement of the decrease in factors' productivity may only result from the estimation of production functions prior to the period of turnaround, which graphs show from 2011 onwards. The methodology is to estimate the production function in the period before 2011 and then calculate from this year what would have been the production with existing factors to be compared to observed production. It raises several problems and in particular the two following ones.

Issues in the specification of production functions

The first is the specification of the production function. Several specifications can be contemplated, more or less flexible that is incorporating more or less degrees of liberty. The less flexible one is the COBB-DOUGLAS production function which incorporates a relatively high (unitary) elasticity of substitution between factors. Effectively estimates on the whole economy and on various sectors exhibit poor results, with in particular the coefficients representing factors' shares most often outside the required interval of $[0,1]$. There is a very abundant literature on this production function which was historically the first to be estimated and it is not necessary to recall it.

At the other extreme is the vintage production function in which the capital is endogenous², estimated along the other parameters of the production function. Several attempts gave unequal results, the main reason being the non-availability of long enough series of investment. Note that this specification, in addition to its flexibility, is particularly convenient for taking into account several factors of production, for instance energy coupled with capital and labor.

Intermediate and thoroughly implemented in econometric work is the Constant Elasticity of Substitution (CES for short) production function in which the related parameter, though "constant" i.e. holding everywhere, is endogenous and estimated along the other parameters, in particular technical progress incorporated in the factors labor and capital. CES is the paradigm of second order "Flexible Functional Forms". A major contribution in this field is the article by Arrow, Chenery, Minhas and Solow known by the acronym SMAC³.

² More precisely its law of accumulation from the series of investments. For a detailed presentation see Alain Bernard, « Agrégation temporelle des fonctions de production à générations de capital », Cahiers du Gama Vol VI, N° 1, février 1985.

³ Arrow, K.J., H.B. Chenery, B.S. Minhas, and R.M. Solow. 1961. Capital-labor substitution and economic efficiency. *Review of Economics and Statistics* 63 (3): 225-247.

For a whole presentation of production functions see Dale W. Jorgenson in *The New Palgrave Dictionary of Economics*.

The second issue is the period on which the estimation must be implemented. The various stages of development experienced by the Tunisian economy as described in the introduction plead for a non-too long period. Various attempts show that the best and most reliable results were for the period 1995 to 2010, which has been selected⁴. Variants on longer periods, starting from 1975 or 1990, have been performed, as well as estimates going beyond 2010.

Specification of the CES production function and estimation

The structural form writes:

$$Q = \{a [L \exp(gt)]^{(-m)} + b [K \exp(pt)]^{(-m)}\}^{(-1/m)}$$

with:

Q : production (here added-value at constant prices)

L : Labor

K: stock of capital

g : technical progress incorporated in Labor

p : technical progress incorporated in Capital

The elasticity of substitution is represented by the parameter m: more precisely it is defined by the formula:

$$s = 1/(1+m)$$

The estimation can be implemented directly on the structural specification which is nonlinear, but usually and more efficiently taking into account the relation between labor demand and its relative price. Assuming that demand for labor adjusts to its marginal productivity it comes:

$$\begin{aligned} \text{Log}(Q/L) &= [-\text{Log } a + mgt + \text{Log}(w/p)] / (1+m) \\ \text{Log}(Q/L) &= s \text{Log}(w/p) + (1-s)gt - s\text{Log } a \end{aligned} \quad (\text{equation 1})$$

with:

p : price of output

w : unit cost of labor

Determination of the remaining parameters b and p is obtained with the relation resulting from the incorporation in the structural specification of the production function of the coefficients estimated with equation 1:

$$\begin{aligned} (Q/K)^{(-m)-a} [(L/K)\exp(gt)]^{(-m)} &= b \exp(-mpt) \\ \text{Log} \{(Q/K)^{(-m)-a} [(L/K)\exp(gt)]^{(-m)}\} &= \text{Log } b - mpt \end{aligned} \quad (\text{equation 2})$$

Assessment and application to the Tunisian economy, globally and by sector

A comprehensive assessment requires a two-fold approach, a global one for the whole economy and a detailed analysis by sector.

Results for the whole economy and interpretation

Econometric estimation of equation 1 over the period 1995-2010 yields the following results: (Students of coefficients are given between brackets)

σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$
0,306	0,014	-1,909
(1,2)	(2,4)	(4,1)
R2		0,982
St. Deviation Regression		0,015

and of equation 2:

⁴ In some cases, the estimation on a longer period revealed more satisfying and was selected.

$\mu\pi$	β	
0,021	1,426	
(4,3)	(32,5)	
R2		0,570
St. Deviation Regression		0,092

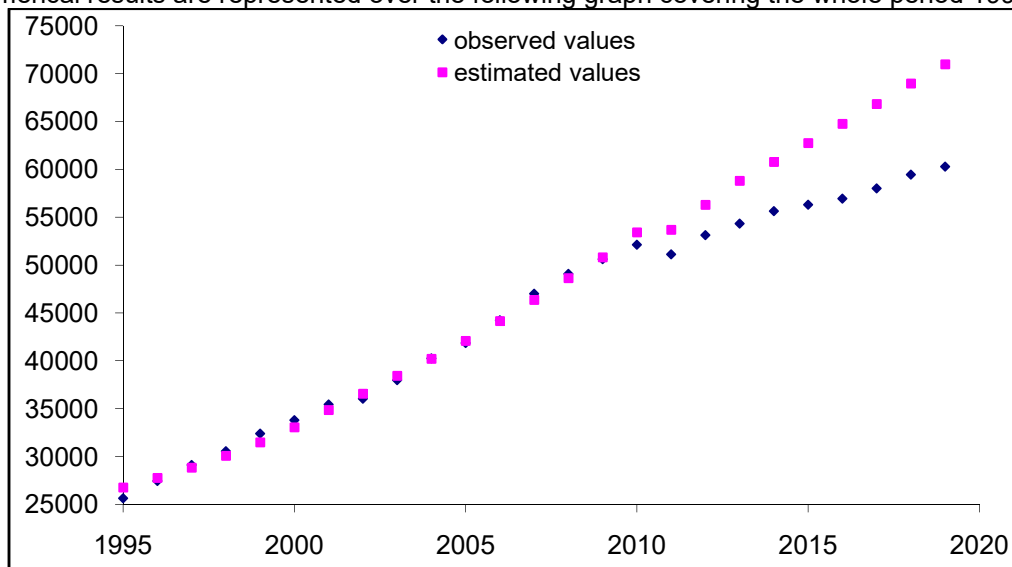
The associated values of technical progress are 2,05% (incorporated in labor) and 0,94% (incorporated in capital). Simulation over the period of estimation gives the following figures, to be compared to observed data:

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	25621	27453	29109	30554	32392	33785	35423	36026	37996	40261	41871	44238	47005	49100	50633	52131
Estimated	26789	27772	28858	30092	31505	33066	34867	36579	38457	40214	42096	44154	46384	48665	50815	53439
Relative gap	4,6%	1,2%	-0,9%	-1,5%	-2,7%	-2,1%	-1,6%	1,5%	1,2%	-0,1%	0,5%	-0,2%	-1,3%	-0,9%	0,4%	2,5%

Extrapolation of the estimated model over the 2010-2019 yields a measure of “potential growth” which can be compared to effective growth:

	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Observed	52131	51132	53126	54352	55654	56315	56935	58016	59467	60299
Estimated	53439	53709	56296	58825	60793	62786	64764	66864	68976	71000
Relative gap	2,5%	5,0%	6,0%	8,2%	9,2%	11,5%	13,8%	15,3%	16,0%	17,7%

The numerical results are represented over the following graph covering the whole period 1995-2019:



The numerical results and in particular the graph exhibit a neat and increasing divergence between effective and “potential growth” of the whole Tunisian economy which reaches 17,7% in 2019. They confirm what was already observed on global data.

The interpretation of this evolution must be cautious: it may reflect a drop in the productivity of the factors of production, or alternately a decrease in their degree of utilization, probably both of them. Unfortunately, it does not exist in the Tunisian statistical system a survey on the

degree of utilization of capital and labor. The fact that the divergence is steadily increasing rather substantiates the first interpretation.

The results obtained here corroborate those of Sawsen Ben Nasr, from the Direction Centrale de la Synthèse et de la Modélisation, “La croissance économique de la Tunisie, Un potentiel décroissant ?” published in the Tribune de l’ITCEQ, N° 23 April 2019.

Results by main sectors

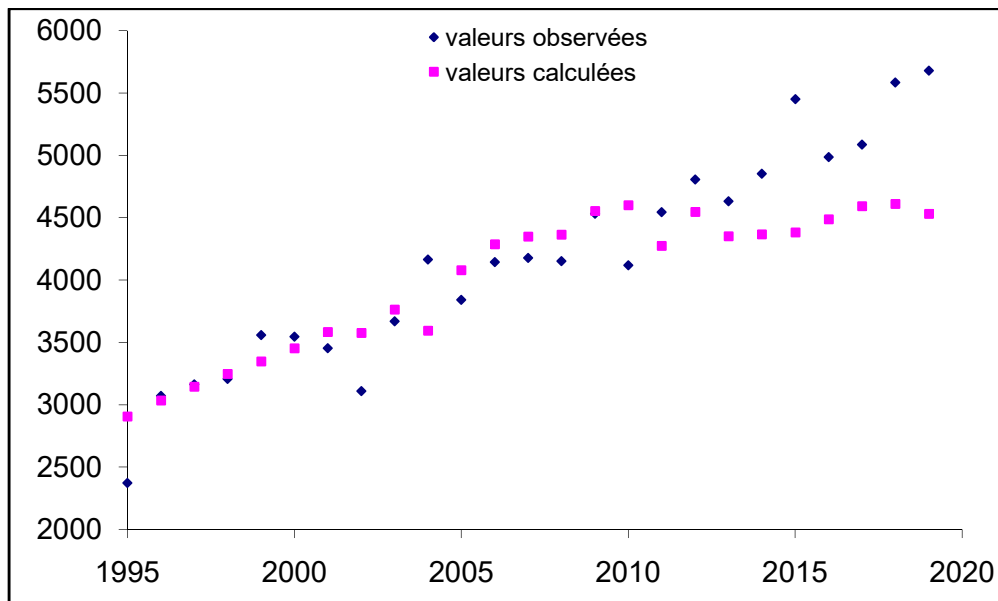
Agriculture and fishing

Equations 1 and 2 yields the values of estimated coefficients, from which can be reckoned the estimates of technical progress incorporated in Labor (1,02%) and in Capital (1,04%)

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,327	0,007	-1,404	0,021	2,030	
(2,4)	(1,2)	(8,5)	(0,4)	(4,8)	
R2		0,604	R2		0,014
St. Deviation Regression		0,081	St. Deviation Regression		0,880

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	2371	3071	3162	3206	3558	3544	3453	3109	3669	4165	3839	4144	4178	4150	4532	4118
Estimated	2904	3034	3144	3247	3347	3452	3583	3575	3764	3593	4079	4288	4348	4365	4555	4602
Relative gap	22,5%	-1,2%	-0,6%	1,3%	-5,9%	-2,6%	3,8%	15,0%	2,6%	-13,7%	6,2%	3,5%	4,1%	5,2%	0,5%	11,7%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	4118	4543	4806	4632	4852	5449	4986	5086	5584	5679						
Estimated	4602	4273	4546	4350	4367	4383	4487	4592	4611	4530						
Relative gap	11,7%	-5,9%	-5,4%	-6,1%	-10,0%	-19,6%	-10,0%	-9,7%	-17,4%	-20,2%						

The estimates confirm what has been observed on global figures in the introduction, i.e. a decrease in the rate of growth of production between the two periods, but which is smaller than the related decreases in the growth of factors. It can be interpreted as a gain in the global productivity of factors or a higher degree of factors’ utilization. Results are represented in the graph below:

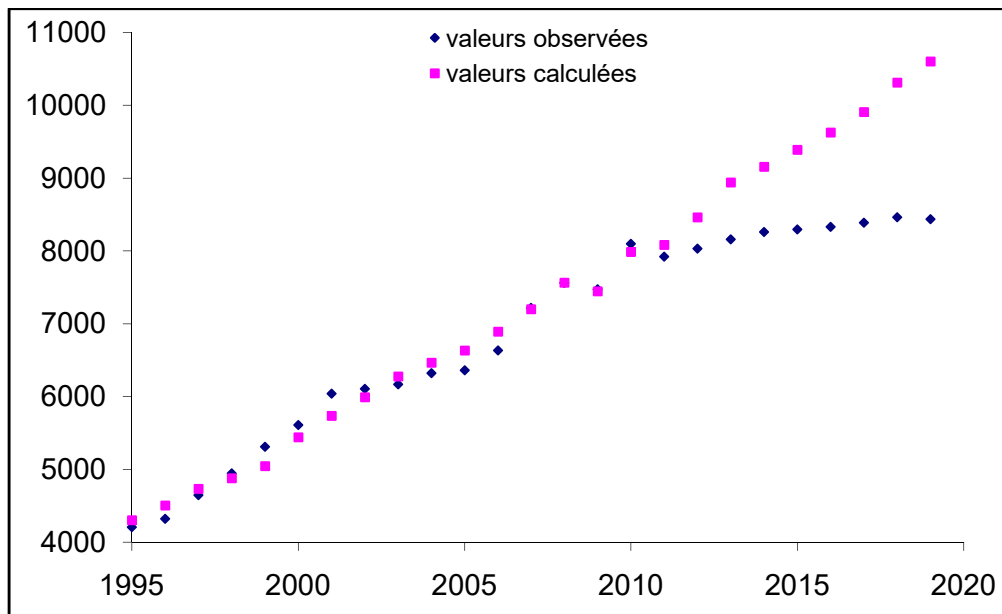


Manufacturing industries

Following the same presentation it comes, with associated estimates of technical progress incorporated in Labor of 2,71% and in Capital of 2,65%:

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,315	0,019	-1,657	0,058	0,381	
(2,0)	(8,0)	(5,6)	(5,2)	(3,9)	
R2		0,955	R2		0,657
St. Deviation Regression		0,025	St. Deviation Regression		0,206

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	4205	4321	4645	4945	5312	5609	6040	6104	6168	6321	6363	6634	7217	7561	7474	8098
Estimated	4302	4506	4734	4877	5047	5442	5738	5992	6279	6467	6636	6894	7201	7564	7449	7987
Relative gap	2,3%	4,3%	1,9%	-1,4%	-5,0%	-3,0%	-5,0%	-1,8%	1,8%	2,3%	4,3%	3,9%	-0,2%	0,0%	-0,3%	-1,4%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	8098	7923	8032	8161	8261	8298	8330	8387	8463	8437						
Estimated	7987	8086	8462	8941	9156	9393	9631	9912	10317	10607						
Relative gap	-1,4%	2,1%	5,4%	9,6%	10,8%	13,2%	15,6%	18,2%	21,9%	25,7%						



The relative evolutions of effective and potential growth are similar to the ones observed for the whole economy, with a divergence which is significantly higher (25.7% compared to 17.7%). Manufacturing industries, though representing a small share of global activity (15.5% in 2010; 14% in 2019), bear an important responsibility in the decrease of the overall productivity of the Tunisian economy.

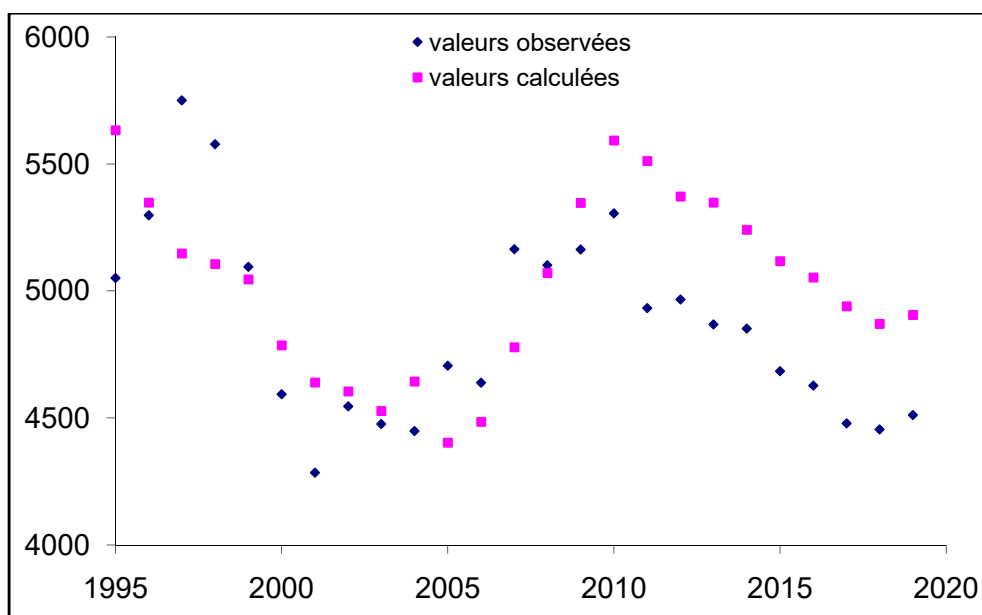
Non-manufacturing industries

In the global landscape of the Tunisian economy, non-manufacturing industries exhibit a very peculiar case. Growth has been particularly irregular, with a decline in production between 1997 and 2004; then a return to growth from 2004 to 2010 and anew a decline from 2010.

Estimation of the production function yields estimates of the coefficients which are statistically significant. The decrease of production during the estimation period is materialized by a negative technical progress of labor productivity (-8.6%).

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,836	-0,014	-1,252	0,005	-0,365	
(4,6)	(3,6)	(3,8)	(4,0)	(35,6)	
R2		0,893	R2		0,537
St. Deviation Regression		0,052	St. Deviation Regression		0,021

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	5051	5298	5749	5578	5095	4593	4284	4545	4476	4448	4705	4638	5164	5101	5163	5305
Estimated	5633	5348	5147	5106	5046	4786	4639	4604	4528	4643	4403	4485	4779	5070	5346	5592
Relative gap	11,5%	1,0%	-10,5%	-8,5%	-1,0%	4,2%	8,3%	1,3%	1,2%	4,4%	-6,4%	-3,3%	-7,5%	-0,6%	3,5%	5,4%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	5305	4932	4966	4868	4851	4684	4627	4479	4454	4511						
Estimated	5592	5512	5371	5348	5241	5117	5053	4940	4870	4905						
Relative gap	5,4%	11,7%	8,2%	9,9%	8,0%	9,2%	9,2%	10,3%	9,3%	8,7%						



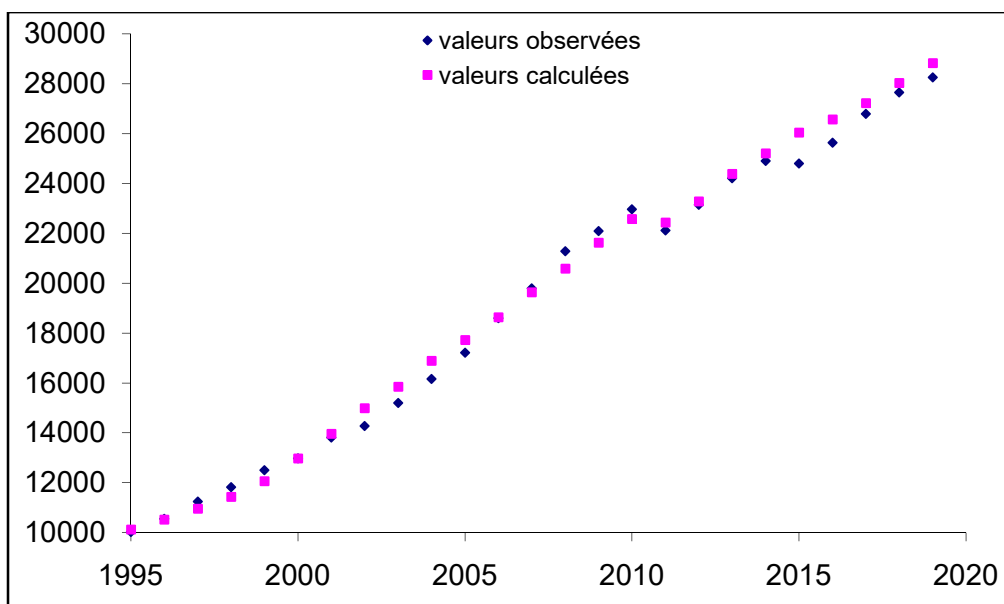
From 2010 effective production remains below potential production of some 10%. Non-manufacturing industries is an aggregate of various and very heterogeneous sub-sectors which deserve to be analyzed separately. This will be implemented in a next section.

Commercial services

Commercial services represent nearly half of the Tunisian economy and an increasing share (44.1% in 2010; 46.9% in 2019). As show the estimates of the production function below, the dynamics of the sector is based on a sustained rate of technical progress incorporated in labor (1,71%) and a growth of labor higher than the average of the economy (1.5% compared to 0.8%).

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,283	0,012	-2,259	0,013	1,156	
(1,3)	(2,7)	(5,2)	(1,8)	(18,8)	
R2		0,897	R2		0,191
St. Deviation Regression		0,031	St. Deviation Regression		0,129

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	10027	10555	11236	11814	12502	12981	13804	14275	15190	16155	17216	18602	19793	21283	22091	22961
Estimated	10125	10524	10959	11433	12065	12969	13960	14989	15856	16888	17728	18638	19642	20590	21632	22577
Relative gap	1,0%	-0,3%	-2,5%	-3,2%	-3,5%	-0,1%	1,1%	5,0%	4,4%	4,5%	3,0%	0,2%	-0,8%	-3,3%	-2,1%	-1,7%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	22961	22110	23143	24214	24902	24795	25638	26792	27649	28257						
Estimated	22577	22437	23281	24391	25213	26050	26573	27230	28040	28830						
Relative gap	-1,7%	1,5%	0,6%	0,7%	1,2%	5,1%	3,6%	1,6%	1,4%	2,0%						



The table and the graph above exhibit an evolution of the effective production that is very close to the potential production, which means that there is no apparent decline in the productivity of factors. But as previously, this has to be broken-up according to the various sub-sectors.

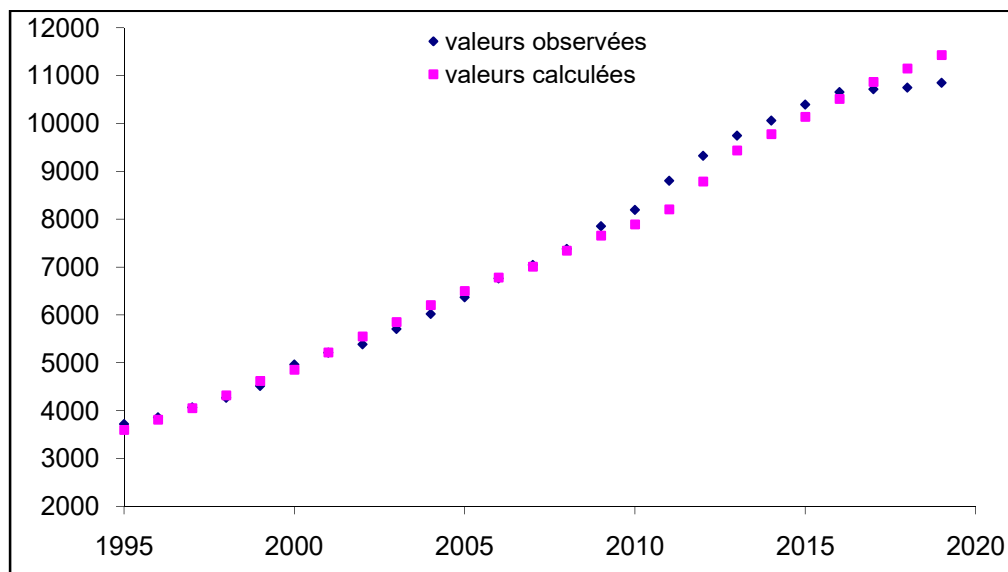
Administration

This sector which represents a growing share of the economy (18.0% in 2019 compared to 15.7% in 2010) differs from other sectors in the sense that production is not directly measured on the market but calculated according to the cost of production.

The results presented below are similar to the case of commercial services. Effective growth and potential growth are fairly close, with a gap limited to 5% in 2019.

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,720	0,012	-0,744	-0,025	-1,660	
(12,2)	(8,7)	(6,2)	(9,9)	(73,1)	
R2		0,998	R2		0,874
St. Deviation Regression		0,007	St. Deviation Regression		0,048

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	3719	3860	4068	4270	4518	4961	5211	5385	5705	6019	6370	6763	7043	7378	7858	8195
Estimated	3602	3812	4051	4322	4623	4860	5220	5551	5852	6211	6500	6783	7010	7348	7659	7892
Relative gap	-3,1%	-1,2%	-0,4%	1,2%	2,3%	-2,0%	0,2%	3,1%	2,6%	3,2%	2,1%	0,3%	-0,5%	-0,4%	-2,5%	-3,7%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	8195	8802	9329	9745	10060	10396	10656	10720	10752	10849						
Estimated	7892	8210	8791	9439	9783	10141	10515	10874	11155	11434						
Relative gap	-3,7%	-6,7%	-5,8%	-3,1%	-2,8%	-2,4%	-1,3%	1,4%	3,7%	5,4%						



On the whole the decline in factors' productivity observed at the aggregate level occurs mainly in industry, both the manufacturing and the non-manufacturing sectors. The latter exhibits, as noted before, a very irregular evolution with periods of growth and periods of decline. The analysis by sub-sectors is presented below, with numerical results given in three appendices.

Analysis by sub-sectors

Manufacturing industries

Among the main sub-sectors, “**industries agricoles et alimentaires**” and “**industries mécaniques et électriques**” show a picture similar to the whole sector, with a production still increasing but at a rate slower than potential growth. The gaps in 2019 are 28.3% for the former -with an inflexion starting in 2015- and 45.9% for the latter. The same can be said for “**Matériaux de Construction Céramique & Verres**” with a production that continued to grow till 2014 and started to decline, well under potential growth (gap of 35.3% in 2019)

“**Industries chimiques**” experienced from 2011 a severe collapse in production, from the range 600 – 700 to the range 450 – 550 while factors of production continued to grow. This is reflected in the data and the graph by a stark decrease of the productivity of factors (minus 87% in 2019).

After a sustained growth the production of “**Textiles, Habillement & Cuirs**” started to decline from the year 2002 with a short period of recovery (2006 to 2008) and again downwards till 2019. In the latter period, factors of production decreased faster than production itself then showing an apparent gain in the productivity of factors.

Finally “**Industries manufacturières diverses**” exhibit from 2010 a slow growth of both production and factors, without any loss in the aggregate productivity of labor and capital.

Non-manufacturing industries

As is the case for manufacturing sectors, non-manufacturing sectors exhibit a large diversity of situations.

On the one hand there are the public utilities, **electricity** and **water**, in which growth of production remains sustained after 2010, close to potential growth for the former, below potential growth for the latter (gap of 22% in 2019).

“**Mines**” experienced from 2010 a severe drop in production while factors, labor and capital, continued to grow and then exhibiting a gap of one to three (potential production approximately three times the effective production). This evolution is particularly disturbing and questions a policy of sustained labor-hiring and investing in a context of declining production.

The sector of “**hydrocarbures**” also experiences a decrease in production which started well before 2010 -with a short recovery during the years 2004 to 2007- but this was associated to a relative stagnation of production factors.

Finally “**Bâtiments & Génie Civil**” exhibits the same picture as the whole economy of a slow growth or stagnant level of production with fast increasing factors of production, with a lag of nearly 50% between effective and potential outputs in 2019.

Commercial services

In the sub-sectors of “**Commerce**” and “**Télécommunications**” the growth of production goes on after 2010 but at a slower rate than during the period 1995-2010 while production factors don’t exhibit any slow-down. The gap between effective and potential outputs in 2019 is 13.2% for the former and 29.8% for the latter.

Activity in “**Hôtels, Cafés & Restaurants**” fell of some 30% during the years 2011 to 2015 but resumed afterwards, rejoining approximately the level of potential production.

In “**Other services**” the growth of output remained very steady while factors of production stagnated, exhibiting a gain in the efficiency of production, which deserves a more detailed examination.

Main teachings and further questions

The Tunisian economy experienced, after a sustained growth till the end of the years two-hundred ten, a severe slow-down at the aggregate level and in nearly all the sectors, agriculture, industry and services and it is important, in order to implement corrective actions, to determine the main reasons of this evolution. There are of course political and social causes but economic factors are also at stake.

A first explanation is a much slower growth of the main factors of production, labor and capital. At the aggregate level lost more than 3 points of growth, from 3.8% to 0.7% for labor; from 5.1% to 1.5% for capital, but the deceleration of production has been much higher, nearly 6 points in the average rate of growth from 2010 to 2019.

A second likely explanation is a decrease in the rate of utilization of the factors of production, resulting from a weaker demand in the domestic and external markets. There is no statistical data which can substantiate this assumption but the fact that the divergence is progressive and long-standing is rather the sign of a slow-down in the productivity of the factors of production. In order to highlight this effect, a methodology has been developed based on the detailed estimation of the production function. Resorting to a CES specification allows to determine all the main parameters at work, the elasticity of substitution between factors and the technical progress incorporated in each of them.

The estimations implemented for the whole economy, the main sectors and sub-sectors give significant and sensible values of the coefficients. Extrapolation of the model beyond the period of estimation, namely between 2010 and 2019, and comparison with observed data allows to measure the “gap” of production, which at the aggregate level represents a loss of growth of around 2%.

Beyond this global result it is interesting to extend the analysis to sectors and sub-sectors. There is an important diversity among sectors, and particularly the industrial ones. What appears is that in the case of several there has been after 2010 and sometimes before a real collapse of the activity which raises questions well beyond the factors’ productivity⁵.

A further issue is the possible contribution of other factors of production, mainly energy which plays an important role in the industrial sector. This is the topic for a new investigation under consideration.

⁵ It will be checked if there are not errors in the series implemented for econometric estimation

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Appendix 1: results by sub-sectors of Manufacturing Industries

Industries agricoles et alimentaires

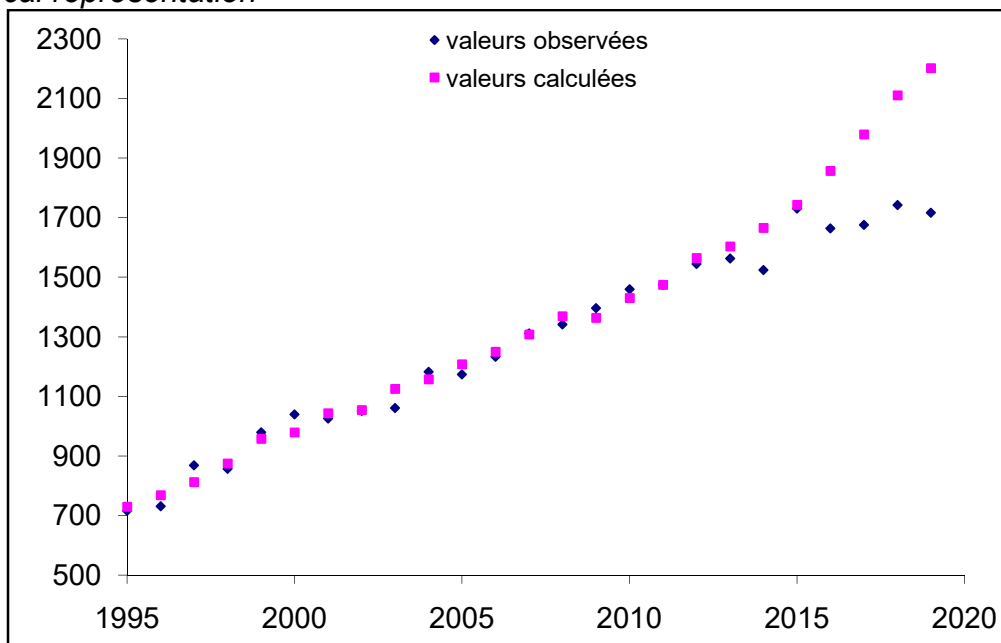
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,334	0,016	-2,012	0,056	0,807	
(2,9)	(4,9)	(8,5)	(6,6)	(10,7)	
R2		0,921	R2		0,755
St. Deviation Regression		0,035	St. Deviation Regression		0,158

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	715	730	868	856	979	1039	1025	1051	1060	1182	1173	1232	1311	1341	1396	1459
Estimated	729	769	813	874	957	979	1043	1055	1126	1157	1207	1250	1308	1369	1364	1430
Relative gap	1,9%	5,3%	-6,4%	2,1%	-2,2%	-5,8%	1,8%	0,3%	6,2%	-2,1%	2,9%	1,4%	-0,2%	2,1%	-2,3%	-1,9%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	1459	1474	1544	1563	1524	1731	1664	1675	1742	1716						
Estimated	1430	1475	1564	1604	1666	1743	1857	1979	2110	2202						
Relative gap	-1,9%	0,0%	1,3%	2,6%	9,3%	0,7%	11,6%	18,2%	21,1%	28,3%						

Graphical representation



Appendix 1continued

Matériaux de Construction Céramique & Verres

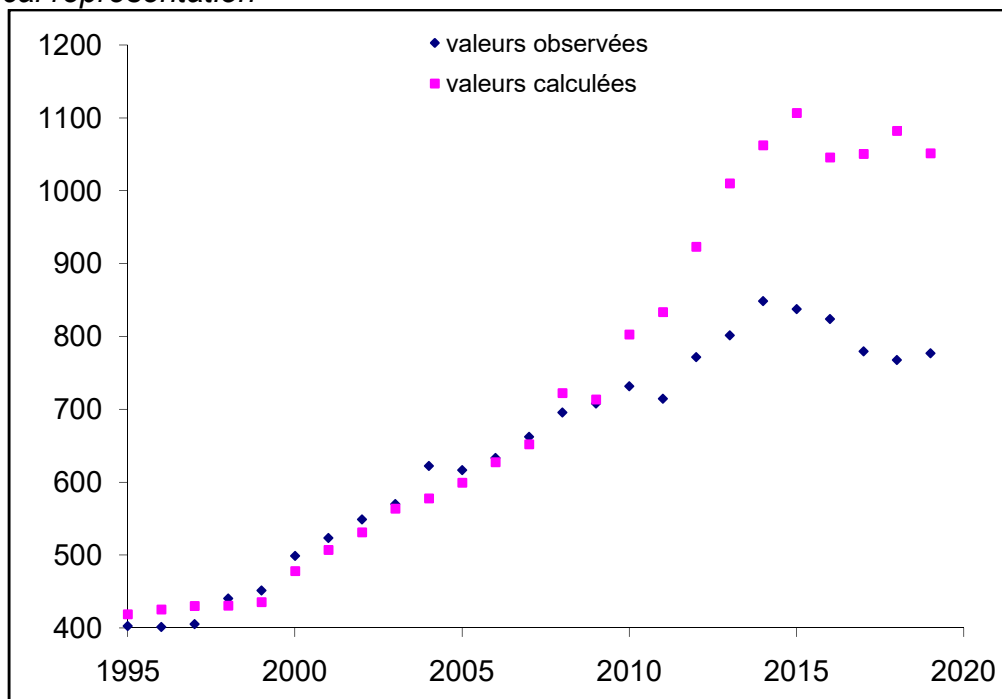
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,517	0,017	-1,526	0,019	0,239	
(2,4)	(2,5)	(3,9)	(3,9)	(5,5)	
R2		0,958	R2		0,522
St. Deviation Regression		0,036	St. Deviation Regression		0,091

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	403	401	405	441	452	499	523	549	570	622	617	633	662	695	708	732
Estimated	419	426	430	431	436	478	507	532	564	578	599	628	652	722	713	803
Relative gap	4,0%	6,0%	6,2%	-2,2%	-3,5%	-4,2%	-3,0%	-3,2%	-1,0%	-7,1%	-2,8%	-0,8%	-1,5%	3,9%	0,8%	9,7%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	732	714	771	802	848	837	824	779	768	777						
Estimated	803	834	923	1010	1062	1107	1045	1051	1082	1052						
Relative gap	9,7%	16,7%	19,7%	26,0%	25,2%	32,2%	26,9%	34,8%	41,0%	35,3%						

Graphical representation



Appendix 1continued

Industries Mécaniques & Électriques

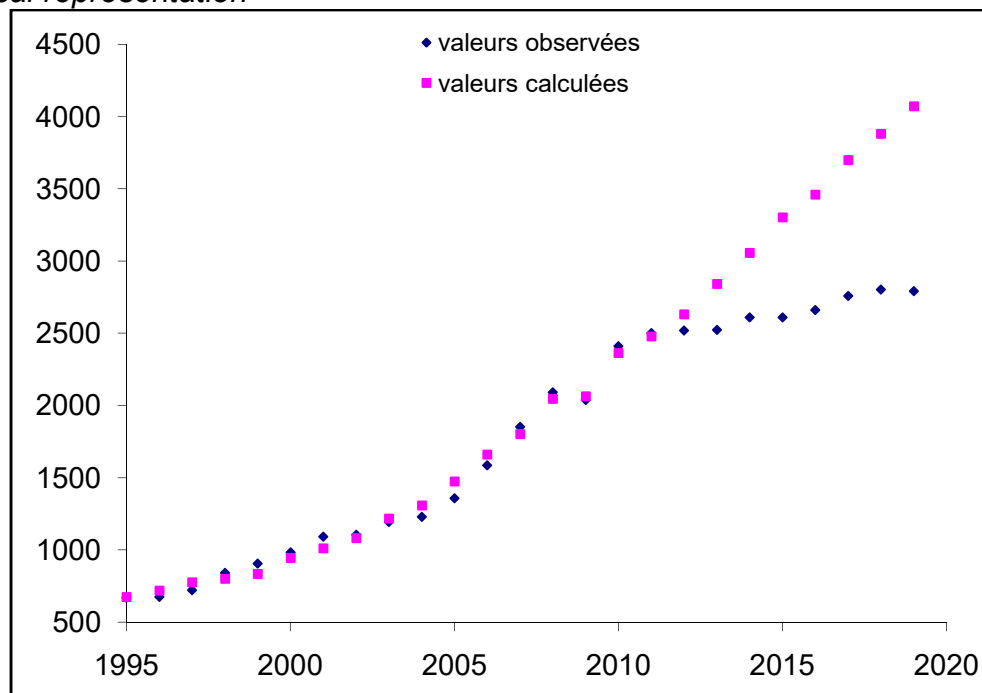
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,564	0,019	-1,262	0,026	-0,849	
(2,0)	(3,0)	(2,0)	(3,2)	(11,8)	
R2		0,911	R2		0,419
St. Deviation Regression		0,049	St. Deviation Regression		0,150

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	670	676	723	842	907	983	1092	1107	1196	1229	1359	1588	1854	2093	2038	2411
Estimated	677	720	778	803	835	945	1013	1085	1218	1310	1476	1662	1804	2049	2066	2364
Relative gap	1,1%	6,6%	7,6%	-4,6%	-7,9%	-3,9%	-7,2%	-2,0%	1,9%	6,6%	8,6%	4,7%	-2,7%	-2,1%	1,3%	-1,9%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	2411	2502	2521	2525	2610	2610	2663	2759	2803	2791						
Estimated	2364	2479	2633	2843	3059	3305	3462	3701	3884	4073						
Relative gap	-1,9%	-0,9%	4,4%	12,6%	17,2%	26,6%	30,0%	34,2%	38,6%	45,9%						

Graphical representation



Appendix 1 continued

Industries chimiques

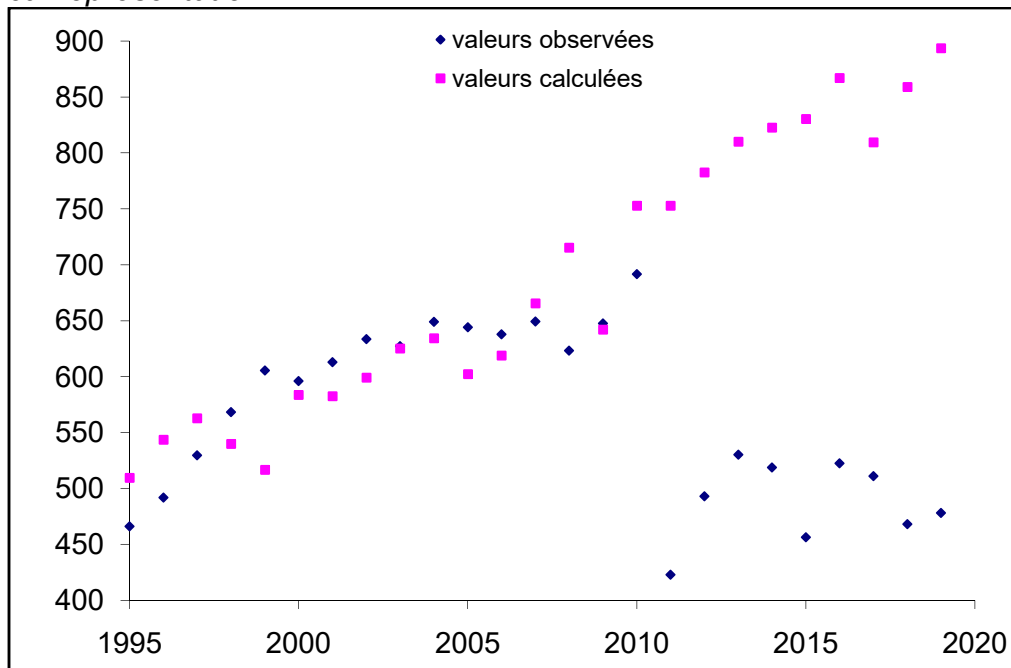
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,817	0,009	-0,826	-0,021	-1,274	
(6,4)	(2,4)	(2,1)	(6,0)	(42,2)	
R2		0,798	R2		0,722
St. Deviation Regression		0,049	St. Deviation Regression		0,063

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	466	492	530	568	605	596	613	634	627	649	644	638	649	623	647	692
Estimated	509	544	563	540	517	584	583	599	625	634	602	619	666	716	642	753
Relative gap	9,3%	10,6%	6,3%	-5,0%	-14,6%	-2,0%	-5,0%	-5,4%	-0,3%	-2,3%	-6,5%	-2,9%	2,5%	14,8%	-0,8%	8,9%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	692	423	493	530	519	456	523	511	468	478						
Estimated	753	753	783	810	823	830	867	810	859	894						
Relative gap	8,9%	78,1%	58,8%	52,8%	58,6%	82,0%	65,9%	58,4%	83,5%	87,0%						

Graphical representation



Appendix 1 continued

Textiles, Habillement & Cuir

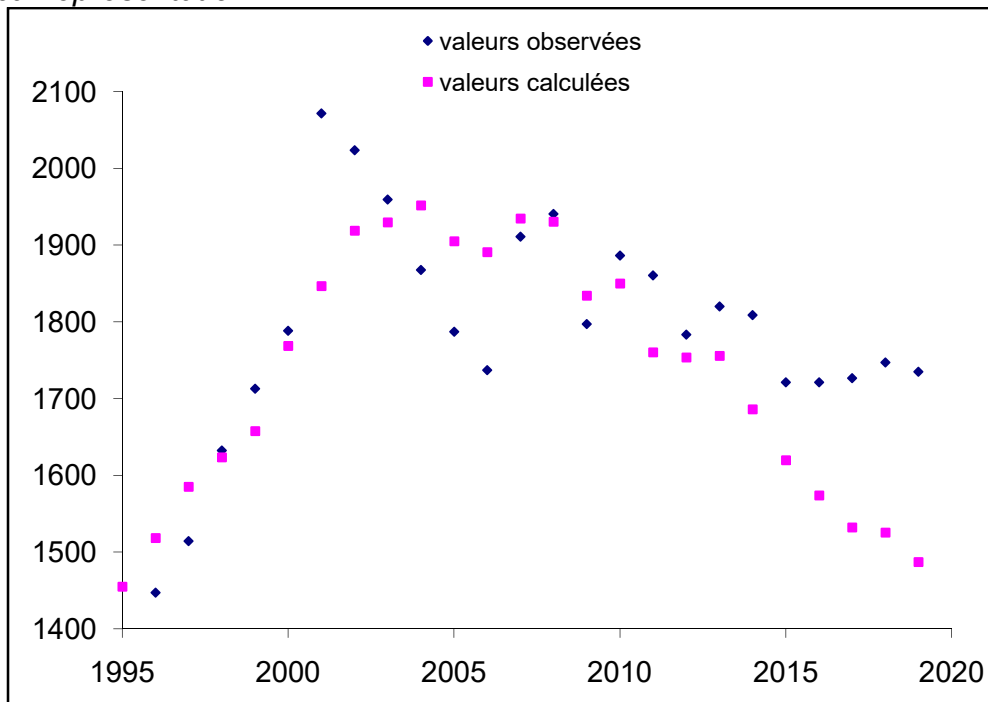
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,354	0,011	-1,296	0,027	-0,442	
(2,4)	(4,7)	(5,3)	(1,2)	(2,3)	
R2		0,645	R2		0,096
St. Deviation Regression		0,042	St. Deviation Regression		0,403

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	1399	1447	1514	1633	1713	1788	2072	2024	1959	1868	1787	1737	1911	1941	1797	1887
Estimated	1455	1518	1585	1624	1658	1769	1847	1919	1930	1952	1905	1891	1935	1931	1834	1850
Relative gap	4,0%	4,9%	4,7%	-0,5%	-3,2%	-1,1%	-10,9%	-5,2%	-1,5%	4,5%	6,6%	8,9%	1,2%	-0,5%	2,1%	-1,9%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	1887	1861	1783	1820	1809	1721	1721	1727	1747	1735						
Estimated	1850	1760	1754	1756	1686	1620	1574	1532	1526	1487						
Relative gap	-1,9%	-5,4%	-1,7%	-3,5%	-6,8%	-5,9%	-8,6%	-11,3%	-12,7%	-14,3%						

Graphical representation



Appendix 1 continued

Industries manufacturières diverses

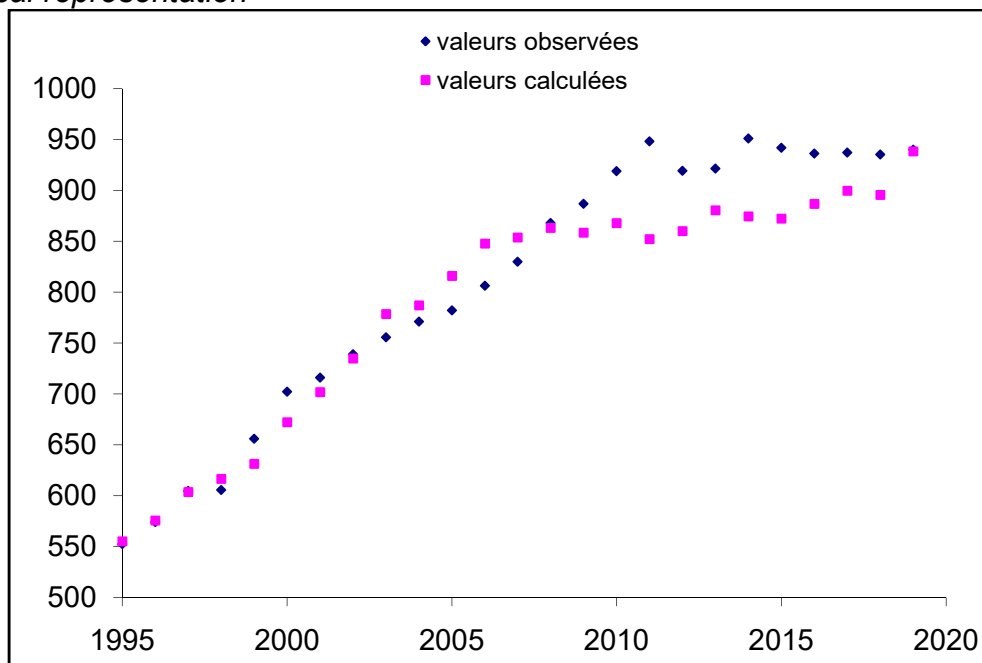
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,232	0,011	-1,770	0,038	1,361	
(3,0)	(9,3)	(16,3)	(3,3)	(13,4)	
R2		0,948	R2		0,441
St. Deviation Regression		0,016	St. Deviation Regression		0,212

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	553	574	605	606	656	702	716	739	755	771	782	806	830	868	887	919
Estimated	555	576	604	617	631	672	702	735	779	787	816	848	854	863	858	868
Relative gap	0,5%	0,3%	-0,2%	1,8%	-3,8%	-4,3%	-2,0%	-0,6%	3,1%	2,1%	4,3%	5,1%	2,9%	-0,6%	-3,2%	-5,5%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	919	948	919	921	951	942	936	937	935	940						
Estimated	868	852	860	880	875	872	887	900	896	938						
Relative gap	-5,5%	-10,1%	-6,4%	-4,4%	-8,0%	-7,4%	-5,3%	-4,0%	-4,2%	-0,1%						

Graphical representation



Appendix 2: results by sub-sectors of Non-manufacturing Industries

Mines

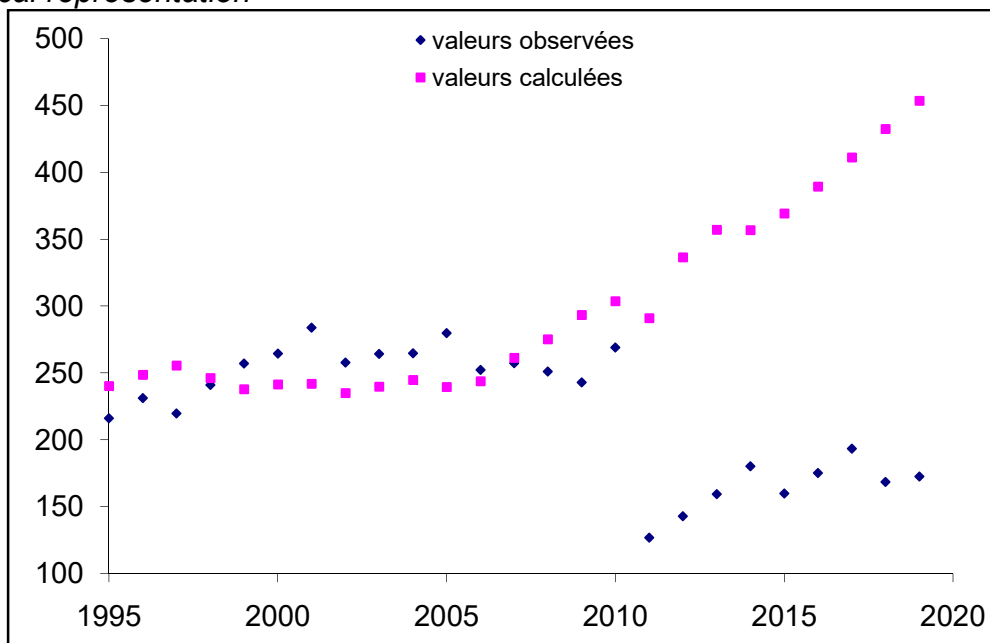
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,223	0,049	-2,552	0,071	3,229	
(2,6)	(12,8)	(12,7)	(2,1)	(10,9)	
R2		0,935	R2		0,241
St. Deviation Regression		0,070	St. Deviation Regression		0,621

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	216	231	220	241	257	264	284	258	264	265	280	252	257	251	243	269
Estimated	240	249	256	246	238	241	242	235	240	245	239	244	261	275	293	304
Relative gap	11,2%	7,6%	16,4%	2,2%	-7,4%	-8,6%	-14,8%	-8,9%	-9,2%	-7,5%	-14,4%	-3,3%	1,5%	9,6%	20,8%	12,9%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	269	127	143	159	180	160	175	193	168	172						
Estimated	304	291	336	357	357	369	389	411	432	454						
Relative gap	12,9%	129,6%	135,8%	123,9%	98,0%	131,3%	122,2%	112,8%	157,0%	162,9%						

Graphical representation



Appendix 2 continued

Hydrocarbures

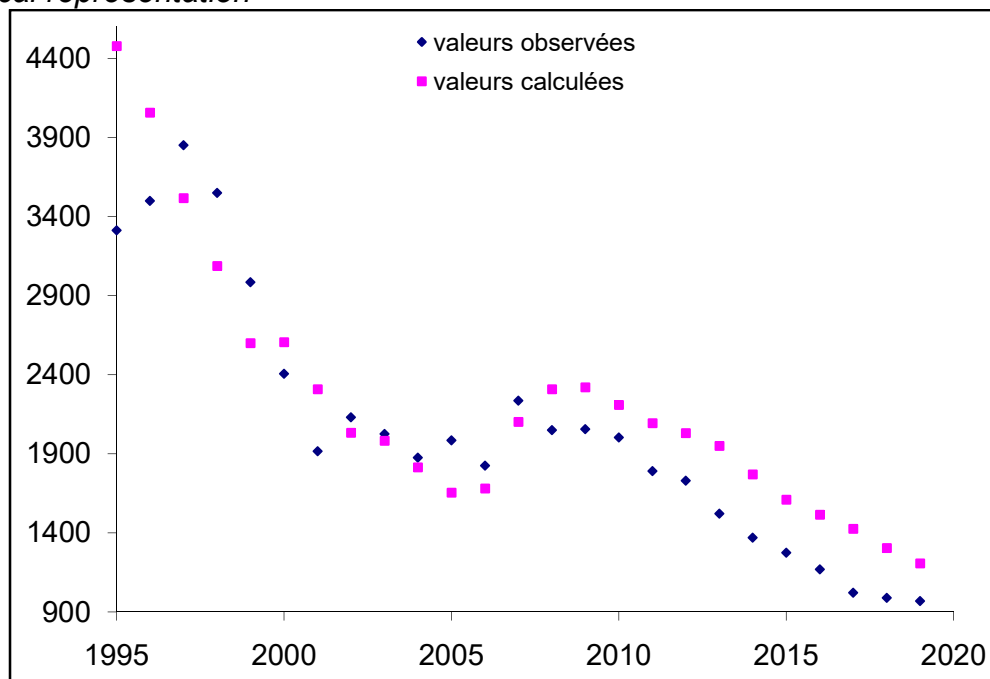
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,377	-0,043	-4,237	-0,139	-0,943	
(2,9)	(3,7)	(5,4)	(4,4)	(3,4)	
R2		0,920	R2		0,582
St. Deviation Regression		0,110	St. Deviation Regression		0,579

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	3313	3498	3851	3549	2985	2405	1916	2131	2025	1875	1985	1824	2236	2050	2055	2002
Estimated	4478	4056	3517	3088	2600	2607	2308	2033	1983	1813	1655	1681	2103	2308	2321	2210
Relative gap	35,2%	16,0%	-8,7%	-13,0%	-12,9%	8,4%	20,5%	-4,6%	-2,1%	-3,3%	-16,7%	-7,8%	-5,9%	12,6%	12,9%	10,4%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	2002	1789	1730	1521	1369	1274	1169	1020	988	968						
Estimated	2210	2094	2030	1950	1771	1609	1515	1425	1304	1207						
Relative gap	10,4%	17,0%	17,4%	28,2%	29,3%	26,3%	29,6%	39,7%	32,0%	24,7%						

Graphical representation



Appendix 2 continued

Electricité

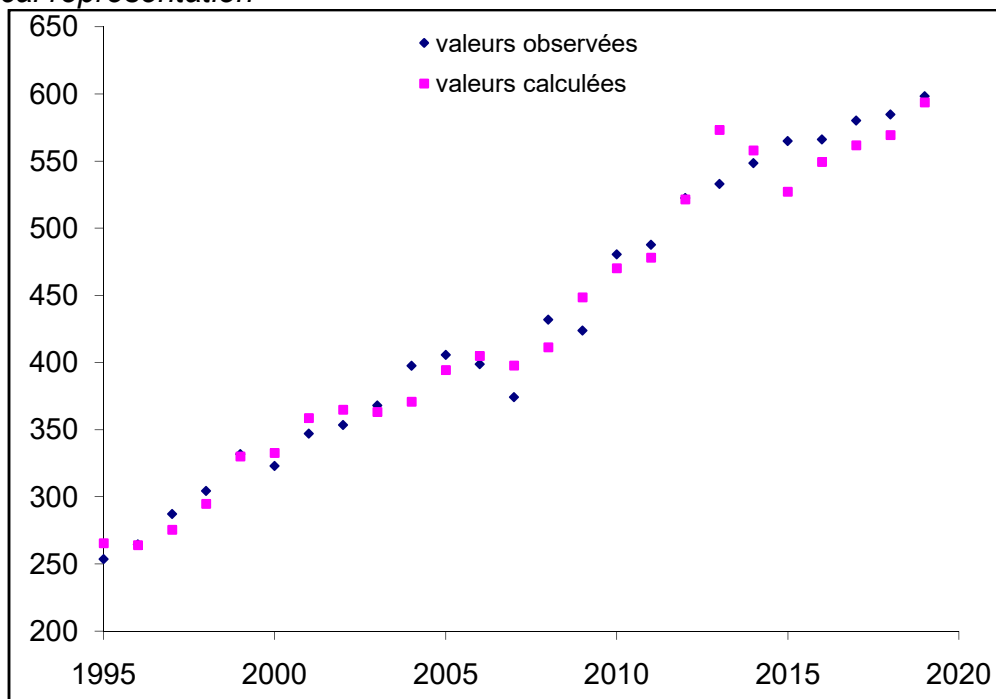
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,389	0,009	-2,436	0,046	3,294	
(2,7)	(1,8)	(7,4)	(8,1)	(66,3)	
R2		0,773	R2		0,823
St. Deviation Regression		0,055	St. Deviation Regression		0,104

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	254	265	287	304	332	323	347	353	368	398	406	399	374	432	424	481
Estimated	265	264	275	295	330	333	359	365	363	371	394	405	398	411	449	470
Relative gap	4,6%	-0,2%	-4,1%	-3,2%	-0,5%	3,1%	3,3%	3,3%	-1,3%	-6,7%	-2,8%	1,6%	6,3%	-4,7%	5,8%	-2,1%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	481	488	522	533	548	565	566	580	585	598						
Estimated	470	478	522	573	558	527	549	562	569	594						
Relative gap	-2,1%	-2,0%	-0,2%	7,5%	1,8%	-6,7%	-2,9%	-3,2%	-2,6%	-0,7%						

Graphical representation



Appendix 2 continued

Eau

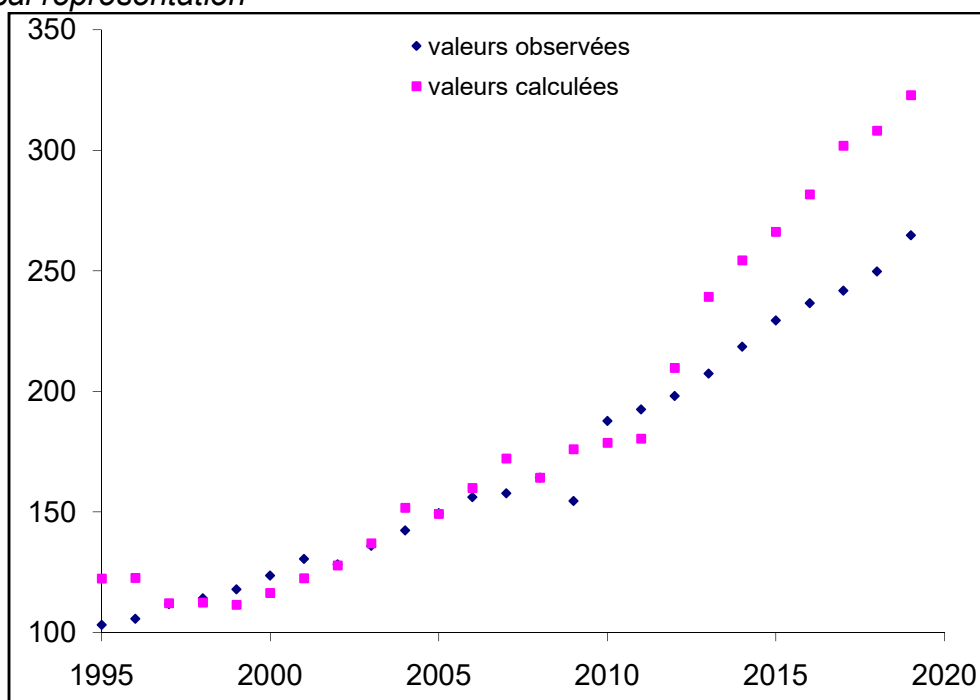
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,139	0,032	-2,336	0,325	14,001	
(0,9)	(1,8)	(11,9)	(6,1)	(29,6)	
R2		0,807	R2		0,724
St. Deviation Regression		0,117	St. Deviation Regression		0,990

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	103	106	112	114	118	124	130	128	136	142	150	156	158	164	155	188
Estimated	122	123	112	112	111	116	123	128	137	152	149	160	172	164	176	179
Relative gap	18,7%	16,1%	0,3%	-1,5%	-5,4%	-5,8%	-6,1%	-0,3%	0,8%	6,6%	-0,2%	2,5%	9,2%	0,1%	13,9%	-4,8%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	188	193	198	207	219	229	237	242	250	265						
Estimated	179	180	210	239	254	266	282	302	308	323						
Relative gap	-4,8%	-6,3%	5,9%	15,4%	16,4%	16,0%	19,1%	24,9%	23,4%	22,0%						

Graphical representation



Appendix 2 continued

Bâtiments & Génie Civil

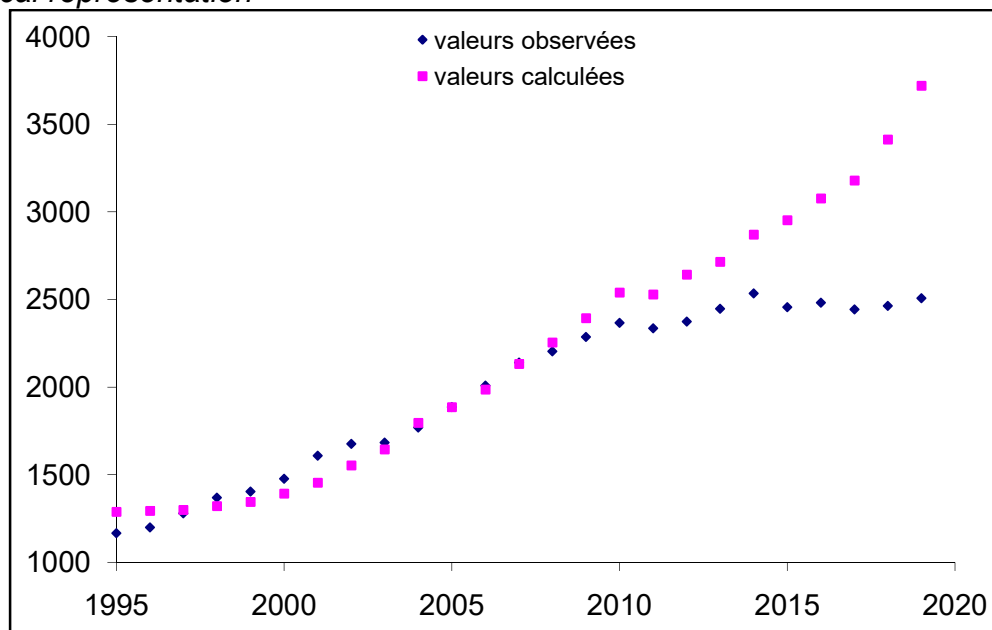
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,099	0,019	-1,405	0,277	-1,764	
(0,8)	(3,1)	(49,2)	(6,5)	(4,7)	
R2		0,806	R2		0,752
St. Deviation Regression		0,057	St. Deviation Regression		0,783

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	1166	1199	1279	1369	1404	1477	1607	1675	1683	1768	1885	2007	2139	2205	2287	2366
Estimated	1289	1294	1300	1321	1344	1392	1454	1553	1644	1797	1885	1986	2132	2256	2393	2541
Relative gap	10,6%	7,9%	1,6%	-3,5%	-4,3%	-5,8%	-9,6%	-7,3%	-2,3%	1,6%	0,0%	-1,1%	-0,3%	2,3%	4,7%	7,4%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	2366	2336	2373	2447	2535	2456	2481	2444	2463	2508						
Estimated	2541	2530	2643	2715	2871	2954	3078	3180	3413	3720						
Relative gap	7,4%	8,3%	11,4%	10,9%	13,2%	20,3%	24,1%	30,1%	38,6%	48,4%						

Graphical representation



Appendix 3: results by sub-sectors of Commercial Services

Commerce

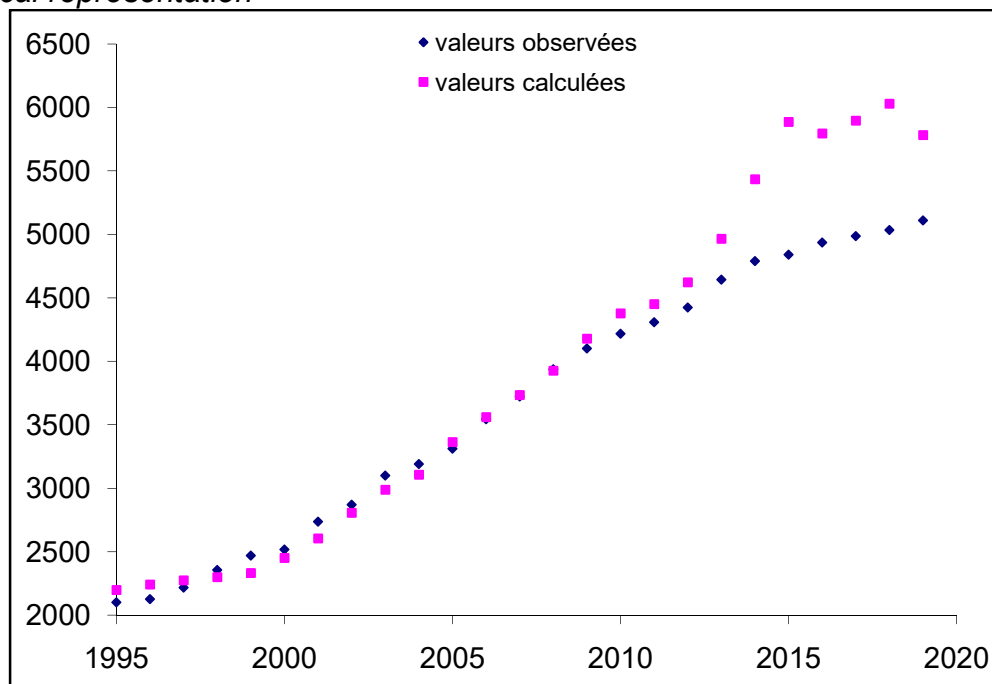
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,259	0,003	-1,843	0,124	-1,843	
(2,3)	(2,2)	(9,2)	(9,0)	(15,1)	
R2		0,468	R2		0,852
St. Deviation Regression		0,029	St. Deviation Regression		0,256

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	2103	2126	2219	2356	2470	2518	2738	2870	3100	3192	3313	3545	3725	3938	4101	4218
Estimated	2199	2243	2274	2300	2333	2453	2607	2808	2989	3108	3366	3563	3735	3929	4180	4380
Relative gap	4,6%	5,5%	2,5%	-2,4%	-5,5%	-2,6%	-4,8%	-2,1%	-3,6%	-2,6%	1,6%	0,5%	0,3%	-0,2%	1,9%	3,9%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	4218	4309	4426	4643	4790	4840	4937	4986	5036	5111						
Estimated	4380	4452	4623	4966	5435	5889	5798	5897	6032	5785						
Relative gap	3,9%	3,3%	4,5%	6,9%	13,5%	21,7%	17,4%	18,3%	19,8%	13,2%						

Graphical representation



Appendix 3 continued

Télécommunications

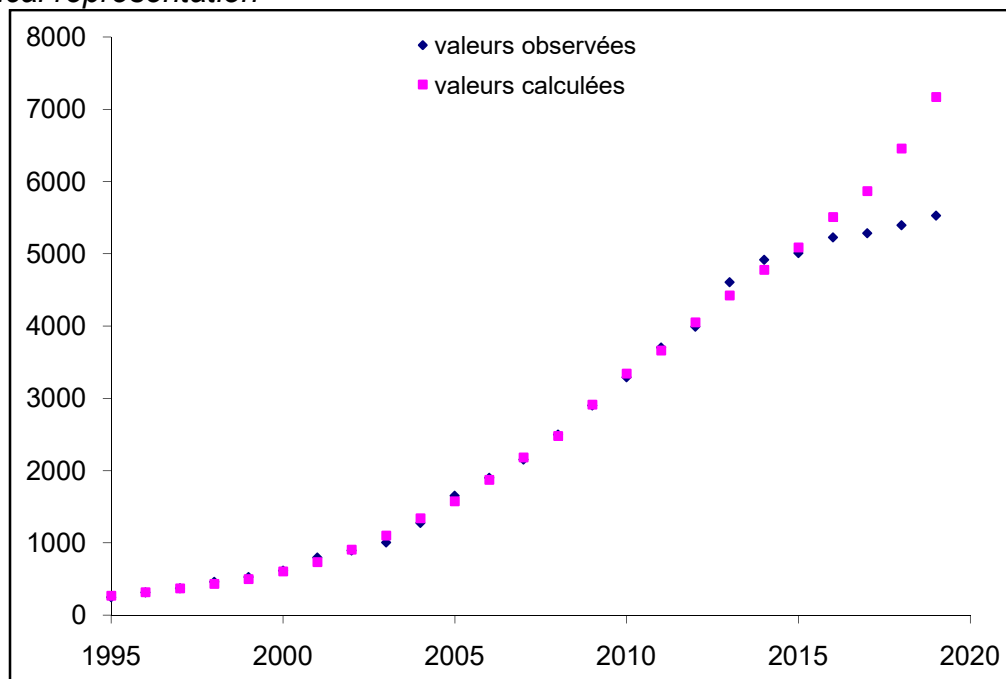
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,356	0,068	-2,253	0,185	2,994	
(3,2)	(5,1)	(11,4)	(25,5)	(46,9)	
R2		0,995	R2		0,979
St. Deviation Regression		0,041	St. Deviation Regression		0,134

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	249	316	374	461	526	617	796	897	1007	1277	1651	1899	2154	2498	2901	3290
Estimated	271	319	373	434	500	605	736	906	1103	1340	1578	1872	2183	2479	2913	3345
Relative gap	9,2%	1,0%	-0,2%	-5,8%	-5,0%	-1,9%	-7,5%	1,1%	9,5%	4,9%	-4,4%	-1,4%	1,3%	-0,8%	0,4%	1,7%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	3290	3701	3991	4604	4915	5009	5224	5281	5392	5527						
Estimated	3345	3663	4054	4424	4779	5090	5507	5868	6457	7172						
Relative gap	1,7%	-1,0%	1,6%	-3,9%	-2,8%	1,6%	5,4%	11,1%	19,7%	29,8%						

Graphical representation



Appendix 3 continued

Hôtels, Cafés & Restaurants

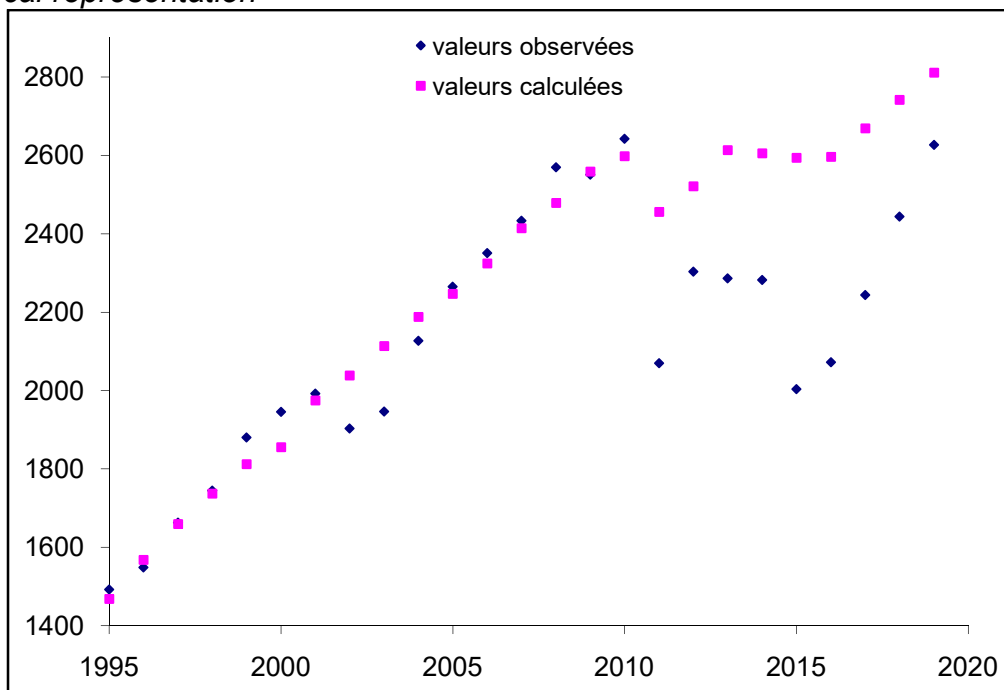
Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,740	0,006	-1,438	0,001	-0,229	
(5,5)	(3,7)	(5,1)	(1,2)	(23,0)	
R2		0,724	R2		0,094
St. Deviation Regression		0,026	St. Deviation Regression		0,021

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	1492	1548	1663	1744	1880	1945	1992	1903	1947	2126	2265	2351	2433	2569	2551	2643
Estimated	1468	1568	1659	1737	1813	1856	1975	2039	2114	2188	2247	2325	2414	2479	2559	2598
Relative gap	-1,6%	1,3%	-0,2%	-0,4%	-3,6%	-4,6%	-0,9%	7,1%	8,6%	2,9%	-0,8%	-1,1%	-0,8%	-3,5%	0,3%	-1,7%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	2643	2070	2303	2286	2282	2004	2072	2244	2444	2627						
Estimated	2598	2456	2521	2613	2605	2594	2597	2669	2742	2811						
Relative gap	-1,7%	18,7%	9,5%	14,3%	14,2%	29,5%	25,3%	19,0%	12,2%	7,0%						

Graphical representation



Appendix 3 continued

AutresServices

Estimation of the production function

Equation 1			Equation 2		
σ	$(1-\sigma)\gamma$	$\sigma \text{Log}(\alpha)$	$\mu\pi$	β	
0,095	0,015	-2,744	-0,203	1,536	
(0,8)	(6,1)	(16,0)	(5,3)	(4,6)	
R2		0,824	R2		0,668
St. Deviation Regression		0,038	St. Deviation Regression		0,705

Comparison of observed and simulated series

	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Observed	3140	3302	3478	3604	3767	3803	4046	4567	5037	5120	5269	5768	6016	6351	6560	6813
Estimated	3129	3205	3283	3448	3573	3912	4332	4743	5075	5557	5763	5951	6119	6173	6136	6096
Relative gap	-0,4%	-2,9%	-5,6%	-4,3%	-5,2%	2,9%	7,1%	3,8%	0,8%	8,5%	9,4%	3,2%	1,7%	-2,8%	-6,5%	-10,5%
	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019						
Observed	6813	6882	6983	7272	7495	7597	7813	8028	8243	8458						
Estimated	6096	5896	5913	5997	6060	6041	6047	5990	5894	5715						
Relative gap	-10,5%	-14,3%	-15,3%	-17,5%	-19,1%	-20,5%	-22,6%	-25,4%	-28,5%	-32,4%						

Graphical representation

